

**THE PHENOMENON OF ELASTIC DEFORMATION OF THE COATING IN THE MEASUREMENT OF TOPOCOMPOSITE HARDNESS BY INSTRUMENTED****N. A. Voronin**

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**DOI: 10.5281/zenodo.3452710****KEYWORDS:** thin coatings, topocomposites, instrumented indentation, hardness, elastic modulus.**ABSTRACT**

In this study a contact problem for an elastic-plastic layered medium is solved. From the analysis of mechanics of the contact interaction of rigid sphere with elastic rigid - plastic layered medium the analytical dependences of the effective hardness on the indentation depth of a rigid tetrahedral pyramid body are received. It is shown that at depths of indentation smaller than the so-called critical indentation depth the origin of plastic deformation occurs only in a coating material. At depths of indentation bigger than the critical indentation depth, the origin of plastic deformation occurs in a base material of a layered body and the composite hardness depends on the coating thickness, the base hardness and effective elastic characteristics of components of the layered medium and does not depend on the coating hardness. In this study the opportunity to define the elastic characteristics of the coating material is proved and the calculation exercise is shown.

**INTRODUCTION**

A significant characteristic of any friction surface functioning is the presence of power loads under mutual displacement in relation to each other. The tribotechnical characteristics of friction surfaces (wear resistance and friction coefficient) are derivative generalized characteristics of specific values of wear resistance and friction forces acting in the actual contact areas. The deformation-strength parameters of the latter are defined by the value of operational loads, physical and mechanical characteristics of the material of the surface layer and geometry of the bodies in contact. It is most reasonable to change directionally the stress-deformation state in the subsurface layer, deformation-strength parameters of the contact areas and the nature of the contact interaction as shown in the history of engineering development by applying coatings and/or modifying the friction surface. Modern technologies of coatings application and modification of the surface layers use composites based on metals, ceramics, and polymeric materials, which permit the application of almost any solid materials from fluorine plastic to diamond and change the structure and composition of the surface layer of structural and instrumental materials [1]. The existing set of traditional and modern technologies permits us to obtain surface layers with changed structure and composition with the thickness from several millimeters or less for thermal vacuum ion-plasma and gas-phase coatings.

The thickness of coatings and layers received by different types of surface treatments cover five orders of magnitude. To specify the surface layer of the material, differing from the base and with the thickness from nanometers to several tens of micrometers, the term "thin coating" is used. Separation of films into thin and thick ones is conventional and relative. The term "thin coatings" is often applied to the surface layers and coatings received by vacuum ion-plasma (VIP) methods, gas-phase methods, sol-gel technology, as well as others. The tribological properties and characteristics of friction with thin coatings are defined mainly by the coating thickness. If the thickness of a tribotechnical coating (modified surface layer) is comparable to the contact area, or is smaller and the ratio of mechanical characteristics of the materials of the layered system components (base and coating) is that external actions are received and localized not only in the coating material but also in the base material, then such subsurface volume represents the surface-layered composite material and is called a topocomposite [2]. This material, as a subject of scientific and engineering interest in western scientific and technical literature, was called surface engineering [3].

The terms of load, the form of bodies in contact, dimensions and mechanical characteristics of the subsurface volume, which include the base material besides the coating material, are decisive while referring to the class of topocomposites and in providing a certain operating capability and tribotechnical characteristics. A correctly formed composite subsurface layer provides to the friction surface and the article in total a technical efficiency



## Global Journal of Engineering Science and Research Management

and economic expediency which the elements of the layered system do not give separately. In most cases the coatings or modified surface layers created by the VIP methods refer to the topocomposites [2]. Precisely these surfaces are the object of analysis and discussion of this work.

Today the surface hardening VIP technologies permit coatings of very different structure and composition to be applied to the operational friction surfaces of products: one-layer homogenous and heterogeneous (gradient), multilayer, composite and combined. In any case, a surface layer of the topocomposite of a complex or simple composition should form a compatible and operable structural system with the base. Any design assumes conducting preliminary calculations at least for the capability to resist the limit loads and, for the contact strength (bearing capacity) of the friction units. It is known that the determination of the stress, the deformation state and deformation-strength parameters of the contact areas for homogeneous solid bodies are possible as a result of solving the contact problems. This is an urgent problem for tribotechnical topocomposites and it is in the sphere of the contact mechanics interaction of solid layered bodies. Some intensity of wear, the appearance of microcracks, flaking and plastic deformation in the layered surface of the body is connecting with the level, type and localization of stresses in the contact area. Although at present, the theory of contact interaction has achieved significant advances, due to mathematical difficulties, some gaps between the theory and practical application of these solution problems appear, particularly for the interaction of layered bodies.

The mechanical characteristics of the subsurface volume are mainly defined by the characteristics of the coating material. But this material does not exist in a compact form. Here the main problem in topocomposite material science should be noted. It is almost a complete absence of reliable reference data on the values of mechanical characteristics of coating materials in the thin-film state and in the form of a thin coating rigidly connected with the base material.

The most suitable method for estimating the mechanical properties of the coating material and effective characteristics of the surface topocomposite is the use of nondestructive certification methods while introducing an indenter of different forms and sizes to the topocomposite surface with simultaneous recording of the implementation diagram during loading and unloading [4]. An important feature of the measured hardness of the strengthened surface is the account of influence of physical-mechanical characteristics of the base material on the amount of hardness. In most researches [5, 6] it has been proved that the measured hardness (microhardness) of thin coatings on the softer base represents composite hardness integrating to different degrees the resistance of plastic deformation of the base material and coating material. As a result, a large number of empirical dependencies were proposed, which can be represented by generalized formalization [6]:

$$H_c = H_0 + \alpha(H_1 - H_0), (1)$$

where  $H_c$  is composite hardness;  $H_0$ ,  $H_1$  – hardness of the base and coating;  $\alpha$  is an empirically determined coefficient.

Numerous studies have shown that formulas similar to dependences (1) do not have a high accuracy to predict the values of composite hardness and hardness of the coating. Therefore, it is necessary to introduce empirical coefficients, the values of which depend on the intuition of the researcher.

When determining the elastic modulus of the coating of a layered system, it is believed that the elastic properties of the base have an impact immediately from the moment of the introduction of the indenter [7]. Acknowledging the combined effects of the coating and the substrate, there were made attempts to develop a function describing the dependence of the change in the composite elastic modulus on the depth of the indenter penetration.

To date, there are many analytical empirical dependencies derived from extrapolation procedures [6]. In the general form the expression for the composite elastic modulus of a layered system, is proposed in a structure similar to composite hardness:

$$E_c = E_0 + (E_1 - E_0)I_0(2)$$



## Global Journal of Engineering Science and Research Management

Here  $I_0 = f(h/a)$ , where  $a$  is the contact radius;  $h$  is the coating thickness; the parameter  $I_0$  is a function that tends to zero if the coating thickness tends to zero and for large values of the coating thickness, it tends to one.

The analysis of the known in the literature solutions of contact problems for the layered systems known in the literature shows that exact solutions are obtained with the application of numerical methods requiring the use of the PC, software and, in some cases, significant expenditures of computer time. Known asymptotic dependencies are applied separately for thick and thin layers. Nevertheless, their use can be wider if we take into account the specific character of the structure of layered systems is considered, as the physical side of solving the problem is no less important than the mathematical.

Based on the analysis of the mathematical model of elastoplastic deformation by a spherical stamp of layered bodies developed by the author, the process of deformation of a topocomposite by a pyramidal indenter during instrumental indentation is described. It is shown, that at the depths of indentation smaller than so-called critical indentation depth of the origin of plastic deformation occurs only in a coating material. At the depths of indentation bigger than critical indentation depth, the origin of plastic deformation occurs in a base material of a layered body and effective hardness depends on coating thickness, base hardness and effective elastic characteristics of components of the layered medium and does not depend on a coating hardness. In this study the opportunity to define the elastic characteristic of coating material is proved and the calculation exercise is shown.

### THEORETICAL PART

Consider a contact interaction of a rigid spherical indenter with a two-layer elastic-plastic half space imitating a solid body with the surface strengthened by a hard coating and representing a surface layer with a thickness of  $h$  rigidly connected with the base thickness  $H > h$  (Fig. 1). We consider the case of implementation of the sphere with radius  $R$ , under normal load  $P$ , without friction and mutual relative displacement of contacting bodies is studied. It is assumed that the materials of a layered system are isotropic, homogeneous and not having residual stress. Friction between spherical indenter and a surface of a layered body was not taken into account. The material of the surface layer had elastic characteristics, the modulus of normal elasticity  $E_1$ , Poisson's ratio  $\mu_1$  and yield point  $\sigma_{T_1}$ , and base material characteristics  $E_0, \mu_0$  and  $\sigma_{T_0}$ . Two types of layered systems with coherent coating connections to the base were analyzed: ideally elastically deformed and elastically rigidly-plastically deformed.

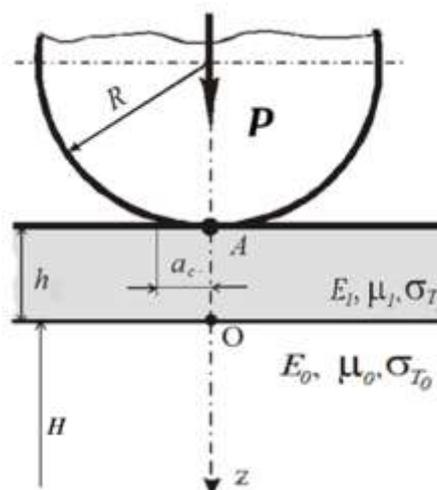


Fig. 1. Calculation scheme for contacting a rigid sphere with an elastic-plastic two-layer half-space (single-layer topocomposite)

For deriving the analytical dependences linking the depth of indentation of a spherical indenter with the mechanical parameters of components of the layered system and the thickness of a top layer, the investigated



## Global Journal of Engineering Science and Research Management

layered system was transformed in a two-layer modeling half-space. The top part of the modeling two-layer half-space represents the top part with thickness  $t_1$  of homogeneous half-space with characteristics identical to that of a bulk coating material. The bottom part of the modeling two-layer half-space represents the bottom part with thickness  $H_c$  ( $H_c = H_0 - t_0$ ) of homogeneous half-space with characteristics identical to that of a base material.

It is known that in the case of loading a homogeneous half-space with a normally distributed load, the displacement of any point inside this half-space is determined by the expression [8]:

$$\delta(x, y, z) = \frac{1}{4\pi G} \left[ 2(1 - \mu)\psi - z \frac{\partial \psi}{\partial z} \right], \quad (3)$$

where  $\frac{1}{G} = \frac{2(1 + \mu)}{E}$ ,  $\psi = \iint_S p(\xi, \eta) \frac{1}{\rho} \partial \xi \partial \eta$  is the potential function of Boussinesq,  $\rho = [(\xi - x)^2 + (\eta - y)^2 + z^2]$  - the current coordinate of any point in the half-space.

For a model layered body, the displacement of a rigid sphere of radius  $R$  can be represented as the sum of the displacement of the base material at depth ( $H_0 - t_0$ ) and the displacement of the material of the surface layer with a thickness  $t_1$  made of the coating material. The analytical expression of such a total displacement of a sphere can be represented as

$$\delta_c = \delta_0 \left[ \theta_0 \frac{1}{\pi} + \left( \frac{K_1}{K_0} \right)^{\frac{2}{3}} \cdot \left( 1 - \frac{1}{\pi} \theta_1 \right) \right], \quad (4)$$

where  $K_0 = \frac{1}{E_0} \mu_0^2$ ,  $K_1 = \frac{1}{E_1} \mu_1^2$  are the effective elastic constants of the single-layer topocomposite components: substrate and coating, respectively;  $K = \frac{K_1}{K_0}$  - the effective contact modulus of a surface layered

$$\text{body, } \theta_0 = f \frac{t_0}{a_0}; \quad \theta_1 = f \frac{t_1}{a_1} = \frac{t_1}{a_0 K^{\frac{1}{3}}}.$$

Subscripts "0", "1" and "c" denote the assignment of the parameter to the base material, coating and layered system.

The expression (4) in the most general form can be represented as

$$\delta_c = \delta_0 \cdot \Phi,$$

where  $\delta_0$  - depth of indentation of rigid sphere in a homogeneous compact material with characteristics of a base material;  $\Phi$  - multiplicand, called an "elastic-geometrical" parameter [9], its range of existence for  $0 \leq \frac{t_0}{a_0} \leq \infty$  is  $1 \leq \Phi \leq K^{\frac{2}{3}}$ ; The parameter  $\Phi$  is analytically exactly determined only in the range of  $h = t_0 \approx 0$

и  $h = t_1 \approx \infty$ .

Representing the expression for  $\Phi$  the power series in the vicinity of the values  $h \approx 0$  and  $h \approx \infty$  and using the Pade two-point approximation method [9], we determine the form of the function  $\Phi$  in the thickness range in the form:



$$\Phi = \sum_{i=0}^m A_i \bar{t}_0^i \left( \sum_{j=0}^n B_j \bar{t}_0^j \right)^{-1}, \quad (5)$$

where  $\bar{t}_0 = \frac{t_0}{a_0}$ ;  $A_i = f(K)$ ;  $B_j = f(K)$ ;  $A_1, A_2, A_3, \dots, A_m$ ;  $B_1, B_2, B_3, \dots, B_n$ - coefficients of two-point Padé

approximants;  $t_0$  - the thickness of a layer with properties of a coating material in the modeling layered two-layer half-space;  $a_0$  - a radius of an impression, considered for the medium with elastic characteristics of a base material at the elastic indentation in it a rigid sphere of radius  $R$  with force  $P$ .

The connection between the geometry of the ideally elastic studied two-layer half-space and the modeling layered medium for all range of thickness of a coating looks like:

$$\frac{h}{a_0} = \frac{t_0}{a_0} \times \Phi^{3/2}. \quad (6)$$

The elastic-geometrical parameter  $\Phi$  depends on geometrical ( $t_0, a_0$ ) and elastic ( $K_0, K_1$ ) characteristics of components of the layered system.

The derivation of the analytical dependences of the indentation depth on the upper-layer thickness  $t_1$  and the procedure for converting the two-layer model half-space into the two-layer half-space being analyzed were outlined in details in [9].

This model yields a series of analytical formulas for the contact deformation and forces and various effective surface characteristics in the case of an ideally elastic and elastic-rigid-plastic two-layer half-space, when a rigid sphere (radius  $R$ ) is introduced in its surface.

1. In the elastic deformation region.

- the depth of indenter introduction:

$$\delta_c = \delta_0 \times \Phi; \quad (7)$$

- the contact radius:

$$a_c = a_0 \times \sqrt{\Phi}; \quad (8)$$

- the pressure at the center of contact area:

$$(p_0)_c = (p_0)_0 \Phi^{-1}; \quad (9)$$

- the effective elastic coefficient:

$$K_c = K_0 \times \Phi^{3/2}; \quad (10)$$

2. At the onset of plastic deformation:

- the limiting depth of indenter introduction:

$$\delta_c^{cr} = \delta_0^{cr} \times \bar{\Phi}; \quad (11)$$

- the limiting contact radius:

$$a_c^{cr} = a_0^{cr} \times \sqrt{\bar{\Phi}}; \quad (12)$$

- the effective yield point:

$$\sigma_{T_c} = \sigma_{T_0} \left( \bar{\Phi} \right)^{1/2} \left( \Phi \right)^{3/2}. \quad (13)$$

The limit elastic-geometrical parameter  $\bar{\Phi}$ , which ranges of existence for  $0 \leq \frac{t_0}{a_0^{cr}} \leq \infty$  is  $1 \leq \bar{\Phi} \leq K^2 Y^2$ .

Parameter  $\bar{\Phi}$  depends on geometrical ( $t_0, a_0^{cr}$ ) and elastic ( $K_0, K_1$ ) characteristics and yield point ( $\sigma_{T_0}, \sigma_{T_1}$ ) of



## Global Journal of Engineering Science and Research Management

the components of a layered system as well.  $a_0^{cr}$  - limit radius of contact (the radius at which indentation of spherical indenter in the surface of homogenous solid results in plastic deformation arises).  $\delta_0^{cr}$  - a depth of limit (critical) indentation of a rigid sphere in a surface of a compact homogeneous material with characteristics of a base material;  $a_0^{cr}$  is calculated for the medium with mechanical characteristics of a base material ;

$$Y = \frac{\sigma_{T_1}}{\sigma_{T_0}}.$$

The limit elastic-geometrical parameter  $\bar{\Phi}$  and connection between geometrical parameters of modeling and the studied two-layer half-space are determined separately for three regions of existence of coating thickness:

- at small thickness

$$\bar{\Phi} = (\Phi)^3; \quad \frac{h}{a_0^{cr}} = \frac{t_0}{a_0^{cr}} \times (\Phi)^{3/2}; \quad (14)$$

- at middle thickness

$$\bar{\Phi} = \frac{0,31}{T_k} (\Phi)^3; \quad \frac{h}{a_0^{cr}} = \frac{t_0}{a_0^{cr}} \times \frac{0,31}{T_k} \times (\Phi)^{3/2}; \quad (15)$$

- at big thickness

$$\bar{\Phi} = (\Phi)^3 \times Y^2; \quad \frac{h}{a_0^{cr}} = \frac{t_0}{a_0^{cr}} \times \frac{(\Phi)^{3/2}}{K^{2/3}}, \quad (16)$$

$$\text{where } T_k = \frac{1}{2} \cdot \left[ \frac{3}{2} \cdot (1 + \bar{t}_0^2)^{-1} - (1 + \mu_0) \cdot \left( 1 - \bar{t}_0 \cdot \arctg\left(\frac{1}{\bar{t}_0}\right) \right) \right], \quad \bar{t}_0 = \frac{t_0}{a_0^{cr}}.$$

The established analytical dependence of an effective yield point for the surface layered material allows calculating a static bearing capacity of such surfaces in contact interaction with a rigid spherical indenter. Bearing capacity of the surface is considered to be the maximum normal load at which plastic deformation of the material on Hertz arises on some depth under contact. For the criterion of transition of elastic deformation in elastic - plastic accept limit depth of indentation  $\delta^{cr}$ .

Taking into account the known equality, connecting the limit size of indentation for a homogeneous material with a yield point, bearing capacity of modeling layered system is determined:

$$P_c = P_0 (\bar{\Phi})^{\frac{1}{2}} (\Phi)^{\frac{3}{2}}. \quad (17)$$

### RESULTS AND DISCUSSION

The dependences of the limiting implementation and carrying capacity on the coating thickness for some layered systems are shown in Fig. 2. There are the limiting amounts of the limiting values of elastic implementation of the rigid sphere to the layered system (at  $h/a_0^{cr}$ ) for some values  $K$  and  $Y$ : 1'-0.5, 3; 2'-0.5, 6; 3'-0.25, 3; 4'-0.25, 6. There are also the results (markers 5) of a numerical calculation of the value of limiting elastic implementation to elastic rigidly plastically medium with parameters  $K = 0.5$ ,  $Y = 3$  for some coating thicknesses. It is clear that for "soft" ( $Y < 1$ ) and "hard" ( $Y > 1$ ) elastic-plastic layered systems the local areas of unusual changes in the values of the limiting implementation and carrying capacity of the thickness of the surface layer are defined. For hard and soft layered systems, the reduction of carrying capacity in the sphere of a thickness of the coating  $h/a_0^{cr} < 1$  is due to the increase in the plasticity of such a layered system (region I in Fig. 2). For hard layered systems with coating thicknesses slightly greater than  $h/a_0^{cr} > 2$  and for soft layered bodies with very small thickness  $h/a_0^{cr} < 0.1$ , there is an increase in the carrying capacity and the value of the limiting implementation is specified (area II in Fig. 2). Since the similar behavior of layered systems takes place in the case when the surface layer lies freely on the base surface, it can be assumed that the observed abnormal behavior of such layered bodies is connected with the structure of the subsurface volume that perceives the load.



Suppose regions *I* and *II* are defined as local areas with respectively decreased and increased the surface strength of layered systems and the presence of such regions as regions with anomalous structural strength.

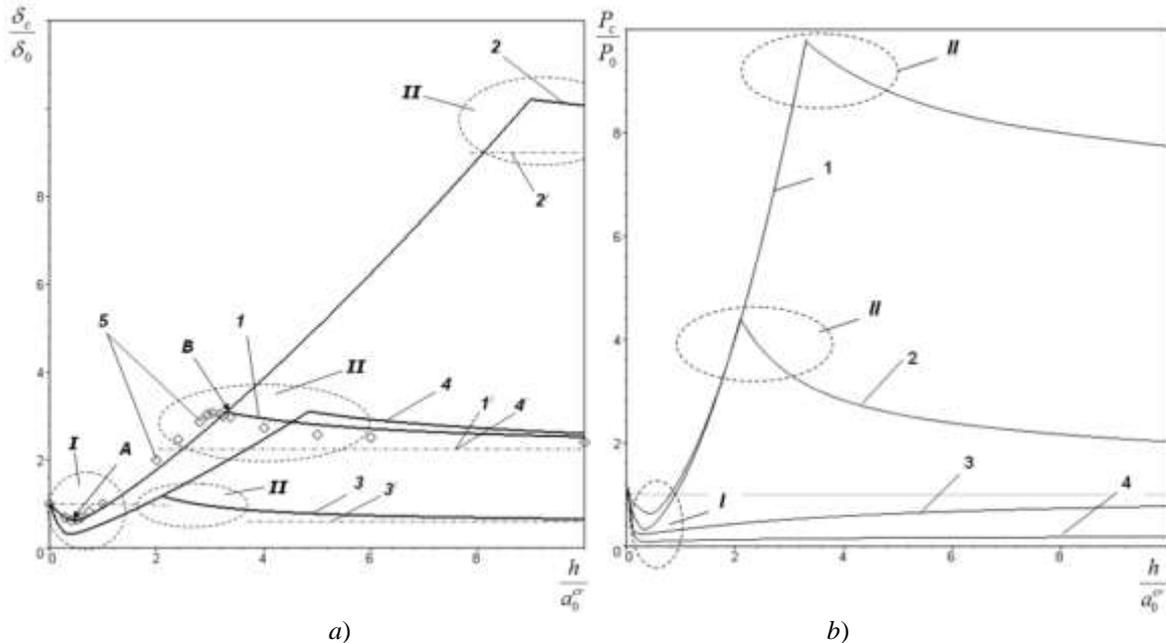


Fig. 2. Dependences of the values of limiting elastic implantation (a) and carrying capacity (b) on the coating thickness for a number of values of elastic-plastic parameters  $K$  and  $Y$ : 1-0.5, 3; 2-0.5, 6; 3-0.25, 3; 4-0.25, 6.

The unusual behavior of the layered systems was confirmed by numerical calculation using the ANSYS software package [10].

From the definition of the surface -layered bodies or topocomposites, being the object of the field of surface engineering, we mention that the topocomposite under contact interaction demonstrates some complex mechanical characteristics of the subsurface volume, which differ from the properties of the coating material and the base material.

For an elastic rigid-plastic layered system with a hard coating, a quantitative criterion determining the classification of a body with thin hardening (harder than the base material) coating to the class of topocomposites can be the critical thickness  $h^*$  of the surficial layer (coating) of the topocomposite. For example, for an elastic-plastic body with elastic-plastic coating, it is a thickness at which the pressure created in the contact area from the applied load leads to an increase in the plastic deformation in the coating material (Fig. 3). While studying the loading of the elastic-plastic surface of the layered body with the rigid sphere (Fig. 3), the order of the value of the critical thickness of the coating is obvious.

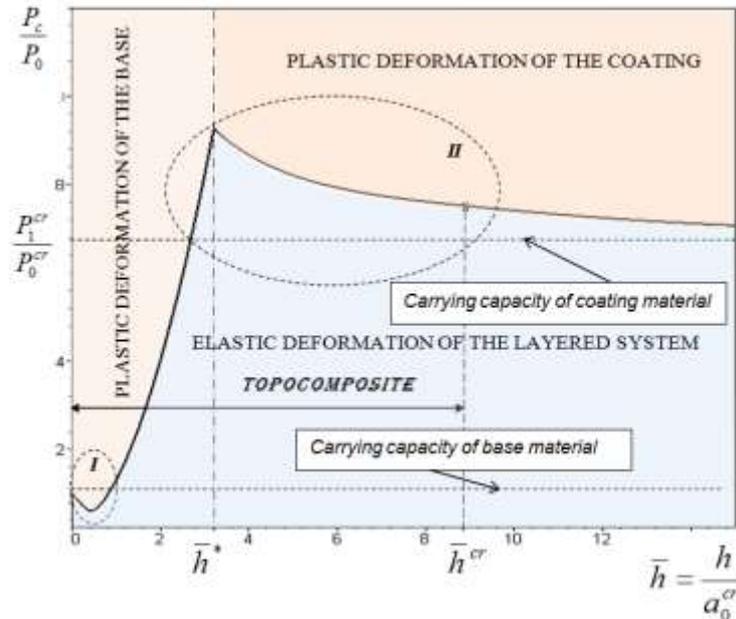


Fig. 3. The summarized dependence of the relative value of the carrying capacity of the surface of the elastic-plastic two-layered solid body while the rigid sphere is implemented into it on the relative thicknesses of the solid coating: I and II-areas of abnormal structural strength

It corresponds to the minimal thickness of the coating  $h^*$  at which plastic deformation appears in the elastic-plastic layered medium in the coating material. Taking into account the effect of structural strength, the boundary of the topocomposite state of the layered elastic-plastic body is realized with coating thicknesses larger than  $h^{cr}$  (Fig. 3). If this thickness is exceeded can be assumed that the base material hardly affects the carrying capacity (contact strength) and tribological characteristics of the friction surface with the coating. If the surface layer (coating) prone to brittle fracture is used in the layered system, the critical thickness will have a different value calculated, for example, on the basis of the condition of the appearance of a critical tension stress in the coating (on the friction surface, at the boundary line or in the coating). The proposed method of determination of the topocomposite state criterion of the layered material permits the identification of precisely the topocomposite as the object of scientific study and engineering application.

In work [11] based on the results of the analysis of the mechanics of contact interaction of a rigid sphere with an elastic rigidly-plastically two-layer half-space imitating the surface strengthened by the hard coating it is shown that for the total possible range of thicknesses of the surface layer there are three characteristic zones of places of origin of the plastic deformation. The results of calculation of the depth  $d_c$  of the location of plastic deformation origin relating to the boundary line "surface layer-base from the coating thickness" are represented in Fig. 4. There are also the diagrams of the limiting implementation and the effective yield point, as well as the stylized image of a two-layer body with a number of thicknesses of the surface layer, for a graphical representation of the location of points of origin of plastic deformation.

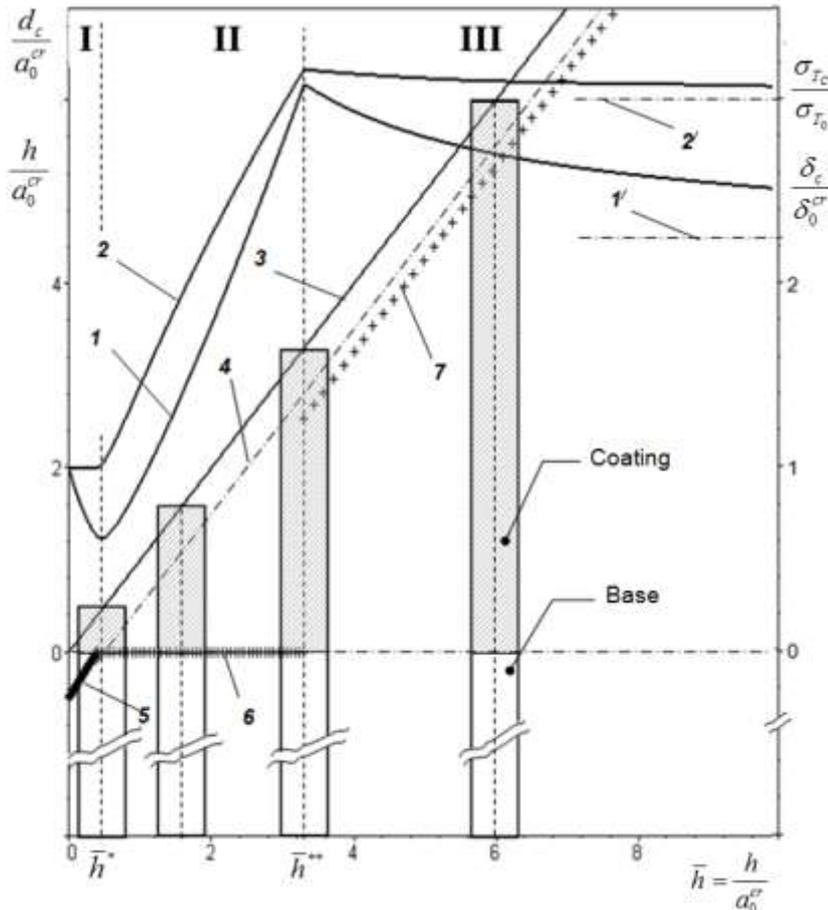


Fig. 4. Diagrams of change: 1-limiting implementation value, 2-effective yield point value, 3-coating thickness, 5, 6, 7-coordinates of origin of plastic deformation for two-layered elastic rigid-plastic medium with parameters  $K = 0.5$  and  $Y = 3$  while implementing of the rigid sphere into it from the coating thickness; 4-location of plastic deformation point of origin in the homogenous solid medium; 1', 2'- limiting amounts of the limiting values of elastic implementation and effective yield point of the layered system, respectively.

It follows from the results that in the first range of thicknesses ( $0 < h < h^*$ ) of coating, the place of origin of plastic deformation is located in the base material, and while the thickness of the surface layer increases it moves from a depth of  $0.5a_0^{cr}$  to the boundary line "coating-base" (curve 5, Fig. 4).

The second range of thickness of coating is located between the values  $h^*$  and  $h^{**}$  and is characterized by the location of the point of generation of plastic deformations in the base material and its permanent location at the boundary line "layer-coating" in the whole second range (curve 6, Fig. 4). The value  $h^{**}$  corresponds to the coating thickness at which the value of the limiting implementation is in the region of the high structural strength of the layered body and has a maximum value. The values of the characteristic thicknesses  $h^*$  and  $h^{**}$  can be determined analytically from the simultaneous solution of the equations describing a change in the value of the limiting implementation from the coating thickness ((7), (14) - (16)). The third range of coating thicknesses begins with the value  $h^{**}$ , covers the whole further possible range of coating thicknesses and is characterized by the location of the point of origin of plastic deformation in the coating material (curve 7, Fig. 4). The feature of the third range of the coating thicknesses is an intermittent transfer of the point of origin of plastic yielding at the value  $h^{**}$  of the layer thickness from the base-coating boundary line in the coating material. In this case the point of origin is first located in the coating material a little deeper than  $0.5a_0^{cr}$  (curve 4, Fig. 4) and while the coating thickness increases it asymptotically tends to this value.



## Global Journal of Engineering Science and Research Management

As shown at Fig. 4 that for the range of coating thicknesses from zero to the value  $h^{**}$  the region where plastic deformation originates is in the base material and only in the third range of coating thicknesses it is in the coating material.

The hardness of compact materials is commonly determined using the indentation of the surface of the studied material by an indenter having one or another shape. It is known that if the indentation of the surface of an elastic-plastic solid by a rigid sphere causes severe plastic deformations of the solid, the average pressure over the indent depends on the yield stress in tension in the following manner [8]:

$$p_m = C \cdot \sigma_T, \quad (18)$$

where  $C$  is the correlation coefficient or the constraint factor. For nonhardenable or ultimate hardened materials (elastically rigidly-plastically media)  $C \sim 2.8 - 3.2$ . For deformation hardenable media, the value of the parameter  $C$  can reach 10 and more [8].

In the practice of measuring the material hardness by using a spherical indenter, the average pressure over the indent within the range of the indent radius  $0.2R \leq a \leq 0.6R$  is the hardness  $H$  of the material being tested.

Thus, analytical expression (18) can be used to describe the dependence of the surface hardness of a layered solid on the surface-layer thickness in the case where elastic rigidly-plastic solids, i.e., nonhardenable or ultimate-hardened materials, are applied as the components of the two-layer medium. Thus, we write the following expression:

$$H_c = H_0 \cdot (\bar{\Phi})^{\frac{1}{2}} \cdot (\Phi)^{-\frac{3}{2}}. \quad (19)$$

Dependence (19) allows us to calculate the theoretical hardness of the surface of a two-layered half-space in the entire possible range of the coating thickness changes using the known values of the hardness and elastic characteristics of the materials of the layered solid components. For the given materials of the components of the layered system, i.e., for the known values of  $Y$  and  $K$ , this analytical expression is a functional dependence; its argument is the coating thickness  $h$  normalizes to the limiting indent radius  $a_0^{cr}$ .

The modern procedures of the standard experimental determination of the hardness or microhardness of a surface covered with a protective coating involve the indentation of the surface by a diamond pyramid and the recording of the load and the corresponding indentation depths or the indent diagonal  $l$ . Simple transformations

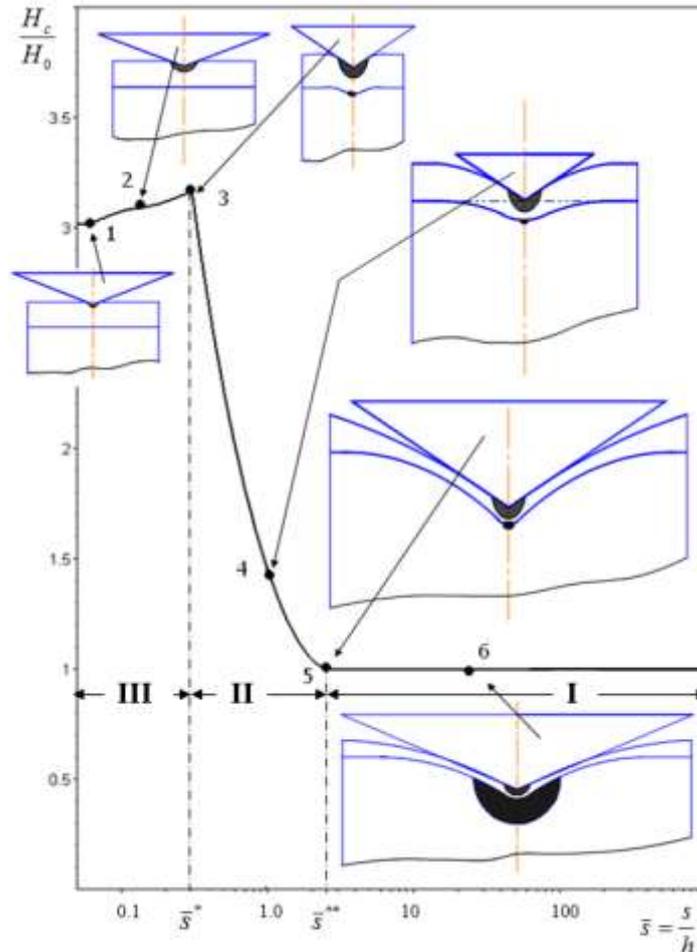
of the above dependences yield the following expression that relates the dimensionless parameter  $\frac{s_c}{h}$  characterizing the depth of indentation of the two-layer half-space by a rigid pyramid (the Vickers pyramid) to

the dimensionless parameter  $\frac{h}{a_0^{cr}}$  characterizing the coating thickness:

$$\frac{a_c^{cr}}{h} = \frac{a_0^{cr}}{h} \sqrt{\bar{\Phi}} \approx 0,4 \frac{s_c}{h}. \quad (20)$$

Thus, expression (19) now represents the analytical dependence of the hardness of the layered system on the depth of indentation of the two-layer elastic rigidly-plastically medium by a rigid tetrahedral pyramid for all values of the thickness  $h$  of the surface layer and ratios of the elastic ( $K$ ) and plastic ( $Y$ ) characteristics of the components of the layered system.

In Fig. 5 the dependence of changes in the theoretical hardness of the surface of the two-layer elastic rigid-plastic medium during the introduction of the rigid pyramidal indenter from the implementation depths  $s_c/h$  is represented.



*Fig. 5. Dependence of varying theoretical hardness of the two-layered elastic rigidly- plastically medium with parameters  $K = 0.5, Y = 3$  on the implementation depth of the rigid four-sided pyramid and schematic representation of the character of the layered system deformation for some implementation depths*

The characteristic points marked in the diagram of varying hardness from the implementation depth (Fig. 5) of the rigid pyramidal indenter, combined with the accepted assumption declaring the elastic rigid-plastically deformation of the layered system, allows us to describe qualitatively and represent systematically the stages of contact interaction of the rigid pyramidal stamp with the layered half-space. In Fig. 5 the abscissa axis shows the values of implementation depths in dimensionless form. The numeric values given on the abscissa axis are shown on a logarithmic scale. The schematic representation of the pyramid contact with the deformed two-layer medium for a number of characteristic moments of contact interaction, without loss of clarity but for a convenient location, are given in Fig. 5 at different scales.

A small implementation depth (Fig.5, point 1) of the pyramidal indenter, plastic deformation arises and develops directly under the indenter top. While the implementation depth increases (Fig. 5, point 2) the area of plastically deformed coating material increases, extending deep and occupying an area similar in form to that of the spherical indenter. When the critical value of the implementation depth  $s^*$  is achieved (Fig. 5, point 3) the area of plastic deformation in the coating material achieves the maximum possible size for this layered system having certain mechanical characteristics (a combination of mechanical properties and geometric parameters of the contact). Elastic deformation also takes place and is related to the total layered system. Therefore, the coating-base boundary line curves slightly bending towards the base material. Further increase of the implementation depth is characterized by a step-like displacement of the plastic deformation area to the base



## Global Journal of Engineering Science and Research Management

material in close proximity to the boundary line. The area of plastic deformation in the coating material is "frozen". With a further growth of the implementation depth the condition for the generation of plastic deformation appears only on the base material at the boundary line at the point located on the contact symmetry axis. While the depth of implementation grows, the diagonal of impression and force loading increases as if in the "self-consistency" mode, which leads to elastic deformation of the layered material and maintenance of the conditions of onset at the point of base-coating sector, but not the development (extension) of plastic flow (Fig. 5, point 4). Such a mode of elastic deformation continues until the indenter reaches the implementation depth  $s^{**}$  (Fig. 5, point 5). From this moment, the base material starts to deform plastically. The area of coating material between two plastic zones continues deforming elastically and, upon further increase of the implementation depth, it remains solid and in the elastic state. For real coatings, it is hardly possible. We know that in the area of the indenter top due to tension stresses the solidity of the coating will be broken. It is often observed experimentally while indenting a hard coating on a soft base. But in our case under the accepted assumption exactly such a state of the material is realized. When we reach implementation depths larger than  $s^{**}$  (Fig. 5, point 6), the area of plastic deformation in the base material grows and is at a significant depth, as between it and the indenter top there is an area of the coating material in the elastic state and the area of plastically deformed coating material. The contribution of elastic deformation of the layered system as compared to plastic deformation of the material at such depths of implementation is insignificant, though the boundary line in the contact area at the depth equal to the coating thickness is considerably bent. Thus it was established theoretically that the mechanism of deformation of the layered system in the second -sector of the diagram of varying layered system hardness is elastic deformation. The result obtained allows us to assume that when the hardness of the surface with the hard coating is changed experimentally is coherently connected with the base; the hardness of the coating material does not affect the value of calculated hardness in the second range of implementation depths.

Analysis of the physical model of contact between the rigid sphere and an elastic two-layer rigidly-plastic half-space or a one-layer elastic rigidly-plastic topocomposite has shown the existence of three typical areas of location of sites where plastic deformation originates for the whole possible range of the surface layer thickness. Analysis of functional dependencies of the effective yield point (13) and hardness (19) in the first and second sectors from the coating thickness (Fig. 4) and implementation depth (Fig. 5) show that neither the effective yield point nor the hardness of the layered system depends on the value of the plastic characteristic of the coating material, i.e., the yield point and hardness of the material, respectively. Only in the third sector the hardness of the coating material act as an argument of the function « $s$ ». Therefore, it is impossible to predict the value and character of a change in the yield point of the material and the hardness of the coating material on the basis of the foregone values of effective yield point and composite hardness of the layered system when referring to the second range of coating thicknesses and implementation depth. It follows from the analysis that the mechanism of deformation of the layered system in the second sector of the diagram of changing layered system hardness is the elastic deformation. The result of the analysis of the mechanism of deformation of the layered system in the second sector of the hardness diagram allows us to assume that, during experimental measurement of the hardness of the surface with the solid coating coherently connected with the base within the depth range larger than  $H_c$ , the hardness of the coating material does not affect the value of the calculated surface hardness. The concept of composite hardness implying a joint effect of the plastic properties of the coating and base materials was traditionally related to this range of implementation depths. However, it is not. The analysis of the functional dependences of the effective yield stress and the hardness on the coating thickness within the first and second ranges and on the indentation depth shows that the substitution of the values of the limiting elastic-geometric parameter  $\bar{\Phi}_{II}$  the expressions (13) and (19) results in the following analytical dependences:

$$\sigma_{T_c}^{II} = \sigma_{T_0} \cdot (\bar{\Phi}_{II})^{\frac{1}{2}} \cdot (\Phi)^{\frac{3}{2}} = \sigma_{T_0} \cdot 0,31 \cdot T_k^{-1}; \quad (21)$$

$$H_c^{II} = H_0 \cdot (\bar{\Phi}_{II})^{\frac{1}{2}} \cdot (\Phi)^{\frac{3}{2}} = H_0 \cdot 0,31 \cdot T_k^{-1}. \quad (22)$$

The coating material hardness is the argument of the function  $H_c$  only within the third range. Therefore, the



## Global Journal of Engineering Science and Research Management

values of the yield stress and hardness of the coating material and the patterns of their variations cannot be predicted based on the known values of the effective yield stress  $\sigma_{T_c}^{\text{II}}$  and the composite hardness  $H_c^{\text{II}}$  of the layered system, which correspond to the second range of the coating thickness and indentation depth. As the coating thickness and the ratio  $K$  of the elastic characteristics of the layered system components are the arguments of the parameter  $T_k$ , it is obvious that the mechanism of the deformation of the layered system within the second portion of the hardness diagram is elastic deformation.

An analytical expression for the loading branch of the diagram of indenter indentation in the case of an indenter with a pyramidal tip and a coated surface was derived in [11]. The derivation is based on the dependence of the composite hardness on the elastic-geometric parameter of a layered body

$$P_c^{\text{II}} = \frac{H_c}{\theta} s_c^2 = \frac{H_0}{\theta} s_c^2 0,31 T_k^{-1}, \quad (23)$$

where the numerical characteristic  $\theta$  depends on the shape of the indenter;  $s_c$  is the depth to which the pyramidal indenter is introduced under a load  $P_c$ .

The dependence of the load on the depth of indenter introduction in an uncoated base may also be expressed in terms of the substrate hardness

$$P_0 = \frac{H_0}{\theta} s_0^2. \quad (24)$$

When  $P_c = P_0$ , taking account of Eqs. (23) and (24), we obtain

$$\frac{s_c^2}{s_0^2} = \frac{T_k}{0,31}, \quad (25)$$

where  $T_k = f(K; \frac{t_0}{a_0^{cr}})$  is a parameter depending on the relative coating thickness and the contact modulus

$$K = \frac{K_1 + K_{in}}{K_0 + K_{in}}, \quad K_{in} = \frac{1}{E_{in}} \mu_{in}^2$$

is the elastic constant of the indenter;  $E_{in}$  and  $\mu_{in}$  are Young's modulus and

Poisson's ratio of the indenter.

The maximum contact radius  $a^{cr}$  of any material is analytically related to the parameter used in measuring the hardness by means of a pyramidal indenter: the depth  $s$  of indenter introduction. For a tetrahedral pyramid with a vertex angle of  $136^\circ$  (a Vickers pyramid),  $s$  is related to the limiting radius  $a^{cr}$  of the impression of a sphere inscribed in a Vickers pyramid as follows:  $a^{cr} = 2.5s$ . Knowing how  $t_0/a_0^{cr}$  is related to the relative thickness  $h/a_0^{cr}$  of the coating and the parameter  $\Phi$ , we may calculate  $T_k$  and determine the dependence of  $(s_c/s_0)^2$  in Eq. (25) on  $K$  and the relative introduction depth  $s/h$  of the pyramidal indenter. In Fig. 6, we plot  $(s_c/s_0)^2$  against  $s/h$  for specific values of  $K$ .

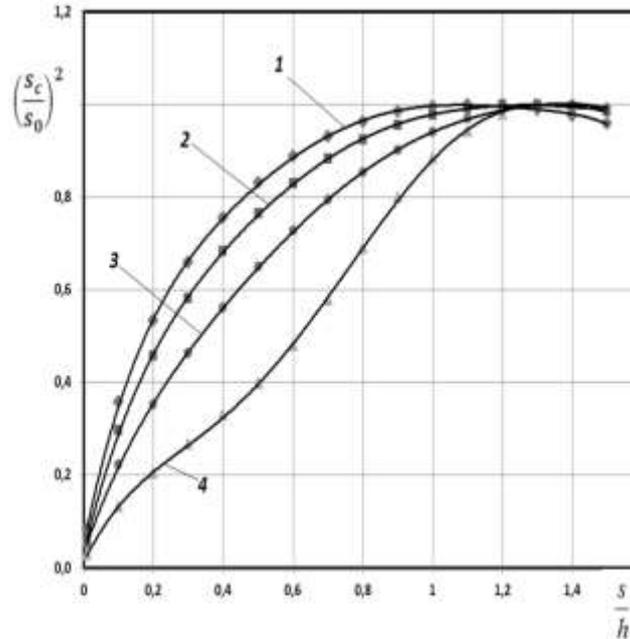


Fig. 6. Calculated curves of  $(s_c/s_0)^2$  as a function of the relative depth of pyramidal indenter introduction, when  $K=1$  (1), 0.75 (2), 0.5 (3) and 0.25 (4).

For a real coated body,  $(s_c/s_0)^2$  may be determined experimentally: by introducing an indenter in the coating surface and also in the substrate surface (with no coating). That permits a comparison of the experimental data with the calculation results and the determination of the desired parameter.

The experimental data are obtained as follows. By means of a microhardness meter (depending on the coating thickness), a diamond tip in the form of a pyramid with a tetragonal (Vickers test) or triangular (Berkovich test) base is pressed into the given layered system (the surface of a part with a thin hard coating), with continuous recording of the loading force and the depth of indenter introduction. A plot of  $P_c$  against  $s_c$  is recorded. The indenter is introduced to a depth less than the coating thickness but always more than 0.1 of that thickness. Analogously,  $P_0$  is plotted against  $s_0$  for an uncoated surface (before coating application or at a special point that is locally free of coating). In Fig. 7, we compare the two curves. On that basis, we calculate for the whole the experimental parameter  $(s_c/s_0)_{exp}$  load range in the test. We also calculate the relative depth of indenter introduction  $(s_c)_i/h$ , where  $h$  is the coating thickness.

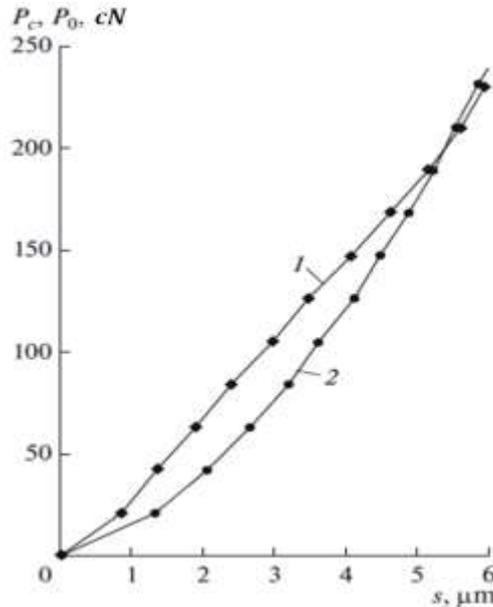


Fig. 7. Dependences of an introduction of an indenter in a layered body (1) and in the surface of the base (2)

We compare the values of  $(s_c/s_0)$  and  $(s_c)/h$  graphically. In Fig. 8, we plot  $(s_c/s_0)$  against  $s_c/h$  and approximate the points by a function such as a «the order polynomial or a spline».

By comparison of the  $(s_c/s_0)$  values with the corresponding theoretical values for various  $K$  values, we may determine the contact elastic modulus  $K$  for the laminar body (Fig. 8). Numerical analysis of the results to minimize the discrepancy between the experimental and calculated results permits refinement of the  $K$  value.

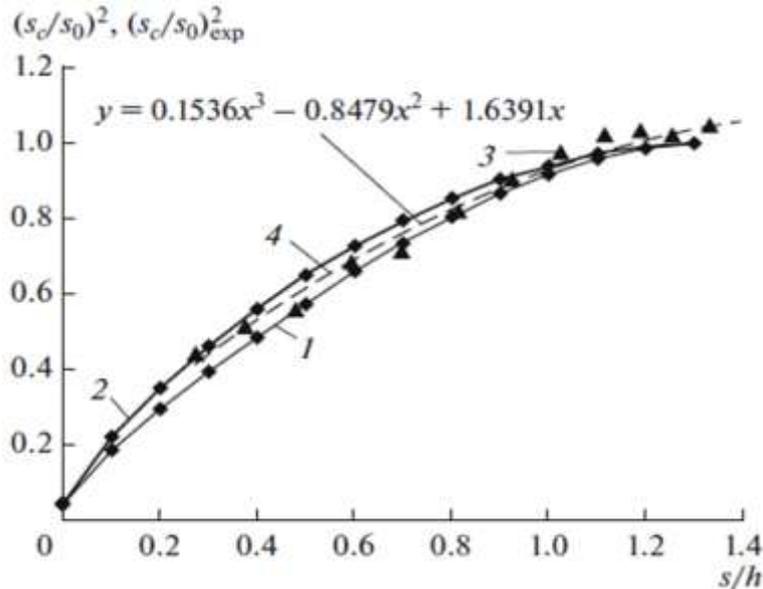


Fig. 8. Dependences of the calculated  $(s_c/s_0)^2$  values for  $K=0.4$  (1) and  $0.5$  (2), the experimental values  $(s_c/s_0)^2$  (3), and the function approximating the experimental data (4).

We compare the experimental and calculated values of  $(s_c/s_0)^2$  over the range  $s/h = 0.2-1.0$ . The reasons for this selection are as follows: when  $s/h = 0-0.2$ , there is a probability of considerable error in measuring the depth of indenter introduction, because the measured quantities are small and because of the error in determining the



## Global Journal of Engineering Science and Research Management

onset of indenter contact with the surface, which is adopted as the coordinate origin. When  $s/h$  approaches 1.0—that is, when the indenter passed through practically the whole coating—there is a high probability that the type of deformation will change, on account of the interface between the coating and the base, which may be regarded as an extended macrodefect, and also the change in the properties of the coating and the base adjacent to the interface (associated with the thermo-chemical processes of coating synthesis).

We may determine Young's modulus of the coating from the expression for the contact elastic modulus  $K$ :

$$E_1 = \frac{1}{K_0 K + K_u (K - 1)} \mu_1^2 \quad (26)$$

The modified Hertz theory can also be used to calculate the elastic modulus of the coating on the basis of the unloading branch of the diagram of indenter introduction. The result obtained by this method is more precise than what the international standard is able to provide [4].

As an illustration of the proposed method, we consider an aluminum nitride coating (thickness 5  $\mu\text{m}$ ) applied by the magnetron method on 12KH18N10T stainless steel. The Young's modulus of the diamond Vickers pyramid is  $E_{in} = 1140$  GPa; its Poisson's ratio is  $\nu = 0.07$ . The elastic characteristics of the base are as follows:  $E_0 = 190$  GPa;  $\mu_0 = 0.3$ . We assume that the Poisson's ratio is the same for the coating and the base. The diagrams of indenter introduction in the base and in the coated surface are recorded on an MTI5 unit, with a maximum load of 2.5 N (Fig. 7).

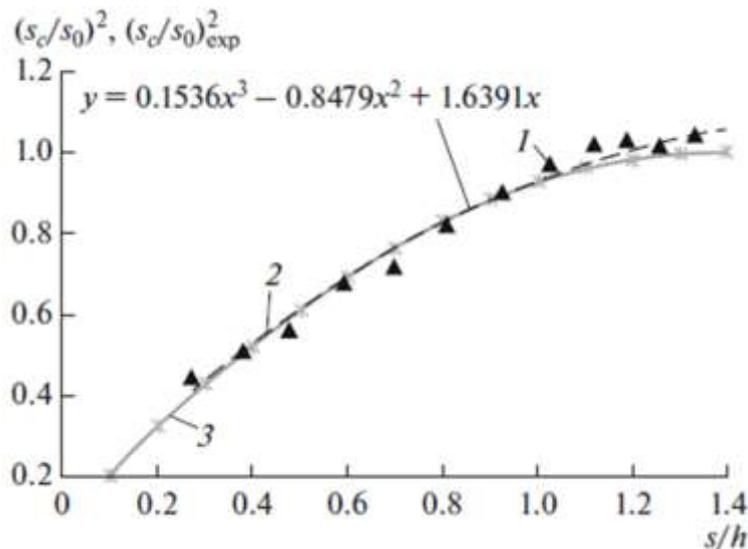


Fig. 9. Comparison of  $(s_c/s_0)^2_{exp}(1, 2)$  and the values of  $(s_c/s_0)^2$  calculated for  $K = 0.45$  (3) as a function of the relative depth  $s/h$  of indenter introduction: (2) polynomial approximation of the experimental data.

In Fig. 8, we show the dependence of  $(s_c/s_0)^2$  on  $s/h$ , as well as the approximating curve  $y = 0.1536x^3 - 0.8479x^2 + 1.6391x$ . The accuracy of the approximation  $R^2 = 0.971$ .

In Fig. 6, we show calculated values of  $(s_c/s_0)^2$  for several values of  $K$ . In Fig. 8, we present a visual comparison of  $(s_c/s_0)^2$  and  $(s_c/s_0)^2_{exp}$ . The correlation between these values is also calculated. It is evident from Fig. 8 that the experimental values  $(s_c/s_0)^2_{exp}$  and their approximating curve lie between the calculated  $(s_c/s_0)^2$  curves for  $K = 0.5$  and  $0.4$ . If we consider the range  $s/h = 0.2-1.0$ , the curve  $(s_c/s_0)^2$  corresponds to  $K$  values of about  $0.42-0.47$ . More precise calculation of  $K$  is possibly by interpolation of intermediate values of  $(s_c/s_0)^2$  in the range  $K = 0.4-0.5$  and assessment of the precision of the correlation between the  $(s_c/s_0)^2_{exp}$  and  $(s_c/s_0)^2$  data sets.

The experimental data best correspond to the calculation results for  $K = 0.45$ , as we see in Fig. 9.

If we calculate the Young's modulus of the coating for  $K = 0.45$ , we obtain  $E_1 = 345.3$  GPa. That is close to the



## Global Journal of Engineering Science and Research Management

values specified in the literature. For *AlN* films applied by reactive magnetron deposition, Young's modulus is about 370 GPa [12]. The *AlN* coatings obtained by gas-phase deposition are characterized by Young's modulus of 332 GPa, in measurements with a Berkovich pyramid [13].

### CONCLUSION

Currently different coatings and modified surface layers are widely used to improve the efficiency of friction surfaces of machine parts and equipment. However the choice of the thickness of a coating and components of topocomposites was carried out empirically, with the help of intuitions and by analogy with the already realized engineering solutions.

In this study, an elastic analysis is presented for the axisymmetric problem of a frictionless rigid sphere indenting a half-space with a thin surface layer. Contact problem has been solved for an ideal elastic two-layered medium and for an elastic-plastic layered medium. Analytical engineering equations for determining the indentation depth, the contact radius, the effective elastic compliance, the effective yield point, and the bearing capacity are obtained. Calculations have been made for film/substrate combinations for which the substrate is either harder or softer than the film and for combinations for which the substrate is either stiffer or more compliant than the film. It is found, unexpectedly, that for topocomposites with thinner coatings the bearing capacity of a topocomposite has a local fall-down for some specific ranges of coating thickness. From the analysis of mechanics of elastic contact interaction of rigid sphere with elastic rigid - plastic two-layer half-space the analytical dependences of the effective hardness of a surface on the thickness of a surface layer (coating) and indentation depth of a rigid tetrahedral pyramid in a surface of the layered body are received. It is found, that at the depths of indentation smaller than so-called critical indentation depth the origin of plastic deformation occurs in a coating material. At the depths of indentation bigger than critical indentation depth, the origin of plastic deformation occurs in a base material of a layered body. Theoretical dependence of effective hardness at the depths of indentation bigger than the critical depth depends on coating thickness, base hardness and effective elastic characteristics of materials of components of the layered medium and does not depend on the hardness of the coating material. Hence, it is impossible to calculate hardness (microhardness, nanohardness) by experimental values of depths of the indentation related to the region of indentation depths bigger than the critical depth. It is shown, that the distinction inelastic characteristics of coating and base materials influence on the value and on the character of the behavior of the hardness considered for a region of depths of indentation smaller than critical depth. It is suggested that the hardness considered for a region of existence of plastic deformation in a coating material only should be named as the effective hardness of a coating, instead of the real one. The real hardness of a coating is the limited value of effective hardness of a coating at the value of indentation depth tending to zero. To define the effective hardness of a coating it is recommended to use the experimental data from the region of the indentation depths related to depths with values smaller than critical depth. It is proved the opportunity to define the elastic characteristic of coating material and the procedure for calculation of the elastic characteristic by experimental data of effective hardness of layered system has been put forward.

Thus, we have proposed a method of calculating the elastic modulus of a thin hard coating from the loading curve in indentation, on the basis of modified Hertz theory. The results may be used in assessing the properties of hardened surfaces.

### ACKNOWLEDGEMENTS

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